Sorting Algorithms

Bubble Sort

- Bubble Sort is the simplest sorting algorithm that works by repeatedly swapping the adjacent elements if they are in wrong order.

- The bubble sort algorithm isn't efficient as its average-case complexity is $O(n^2)$ and worst-case complexity is $O(n^2)$.

First Pass:

(**5 1** 4 2 8) –> (**1 5** 4 2 8), Here, algorithm compares the first two elements, and swaps since 5 > 1.

(1 **5 4** 2 8) –> (1 **4 5** 2 8), Swap since 5 > 4

(1 4 **5 2** 8) –> (1 4 **2 5** 8), Swap since 5 > 2

(1 4 2 **5 8**) –> (1 4 2 **5 8**), Now, since these elements are already in order (8 > 5), algorithm does not swap them.

Second Pass:

(**1 4** 2 5 8) –> (**1 4** 2 5 8)

(1 **4 2** 5 8) –> (1 **2 4** 5 8), Swap since 4 > 2

(1 2 **4 5** 8) –> (1 2 **4 5** 8)

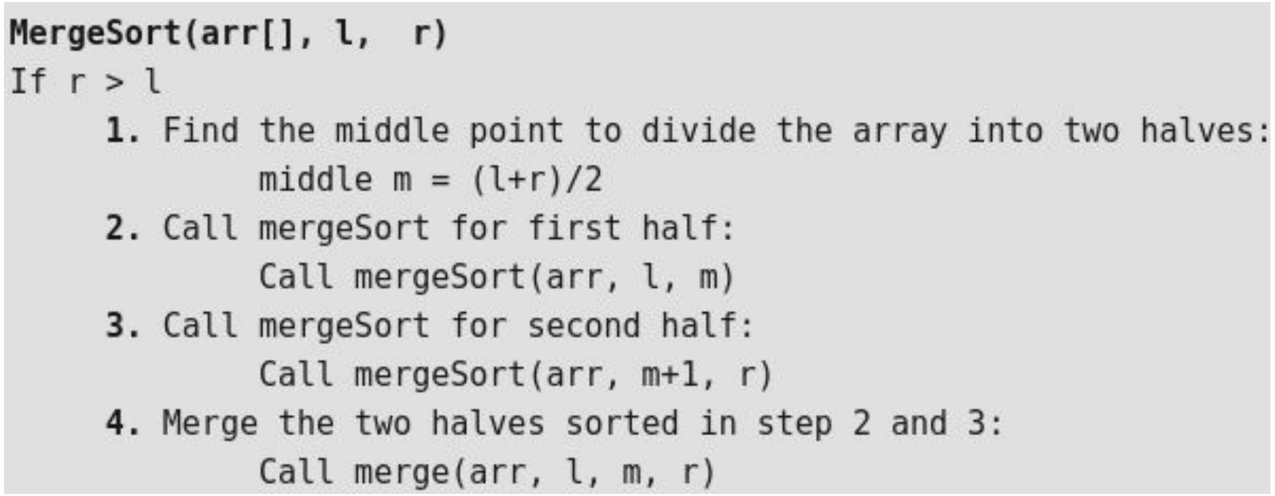
(1 2 4 **5 8**) –> (1 2 4 **5 8**)

Now, the array is already sorted, but our algorithm does not know if it is completed. The algorithm needs one **whole** pass without **any** swap to know it is sorted.

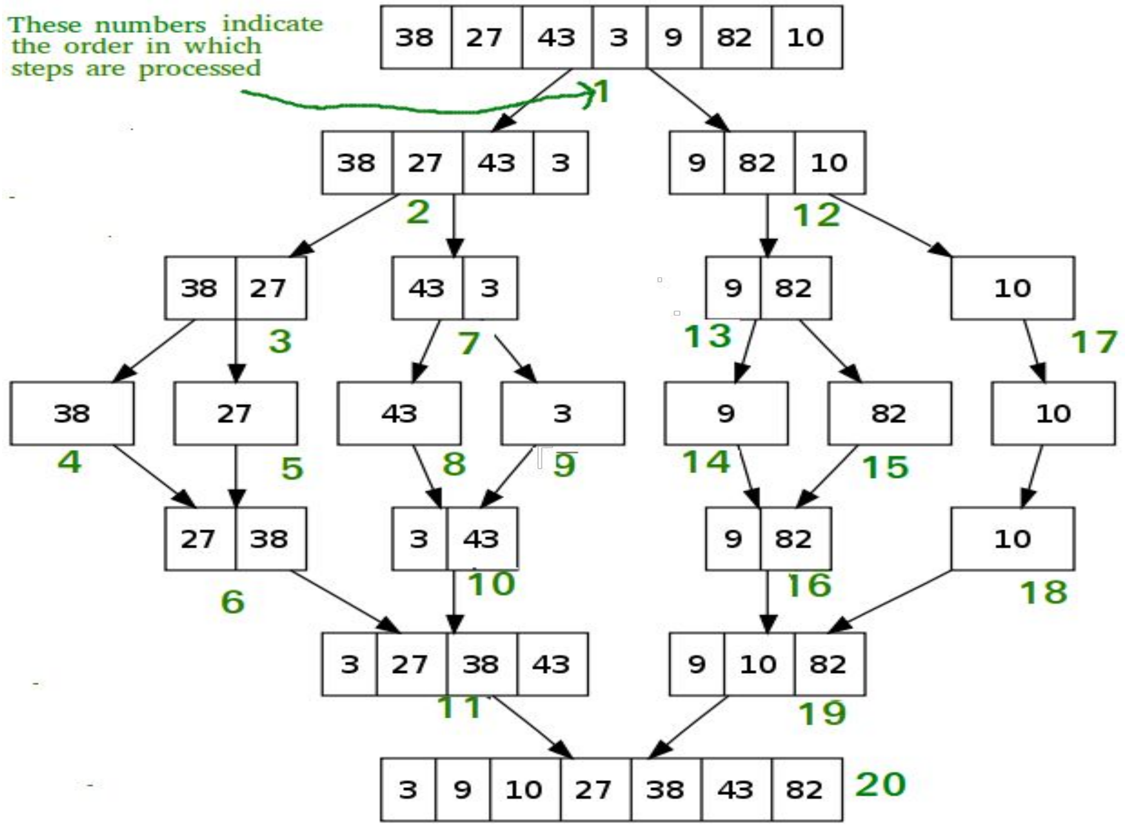
```c
void bubble_sort(long list[], long n)
{
  long c, d, t;

  for (c = 0 ; c < n - 1; c++) {
    for (d = 0 ; d < n - c - 1; d++) {
      if (list[d] > list[d+1]) {
        /* Swapping */
        t         = list[d];
        list[d]   = list[d+1];
        list[d+1] = t;
      }
    }
  }
}
```

Merge Sort

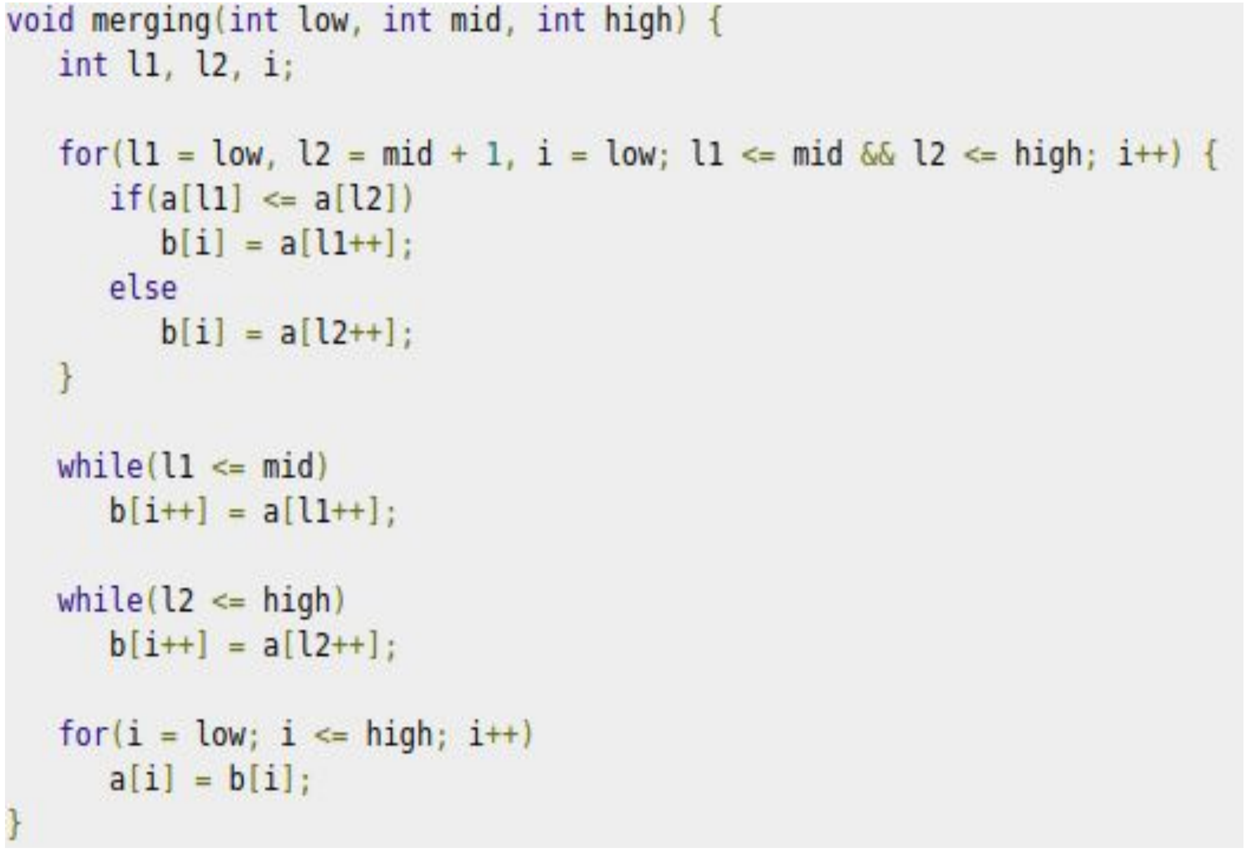
Merge Sort is a Divide and Conquer algorithm. It divides input array in two halves, calls itself for the two halves and then merges the two sorted halves. **The merge() function** is used for merging two halves. The merge(arr, l, m, r) is key process that assumes that arr[l..m] and arr[m+1..r] are sorted and merges the two sorted sub-arrays into one.



```
MergeSort(arr[], l,  r)
If r > l
     1. Find the middle point to divide the array into two halves:
             middle m = (l+r)/2
     2. Call mergeSort for first half:
             Call mergeSort(arr, l, m)
     3. Call mergeSort for second half:
             Call mergeSort(arr, m+1, r)
     4. Merge the two halves sorted in step 2 and 3:
             Call merge(arr, l, m, r)
```


These numbers indicate the order in which steps are processed
38 27 43 3 9 82 10
1
38 27 43 3
9 82 10
2
12
38 27
43 3
9 82
10
3
7
13
17
38
27
43
3
9
82
10
4
5
8
9
14
15
27 38
3 43
9 82
10
6
10
16
18
3 27 38 43
9 10 82
11
19
3 9 10 27 38 43 82
20

```c
void sort(int low, int high) {
   int mid;

   if(low < high) {
      mid = (low + high) / 2;
      sort(low, mid);
      sort(mid+1, high);
      merging(low, mid, high);
   } else {
      return;
   }
}

int main() {
   int i;

   printf("List before sorting\n");

   for(i = 0; i <= max; i++)
      printf("%d ", a[i]);

   sort(0, max);

   printf("\nList after sorting\n");

   for(i = 0; i <= max; i++)
      printf("%d ", a[i]);
}
```



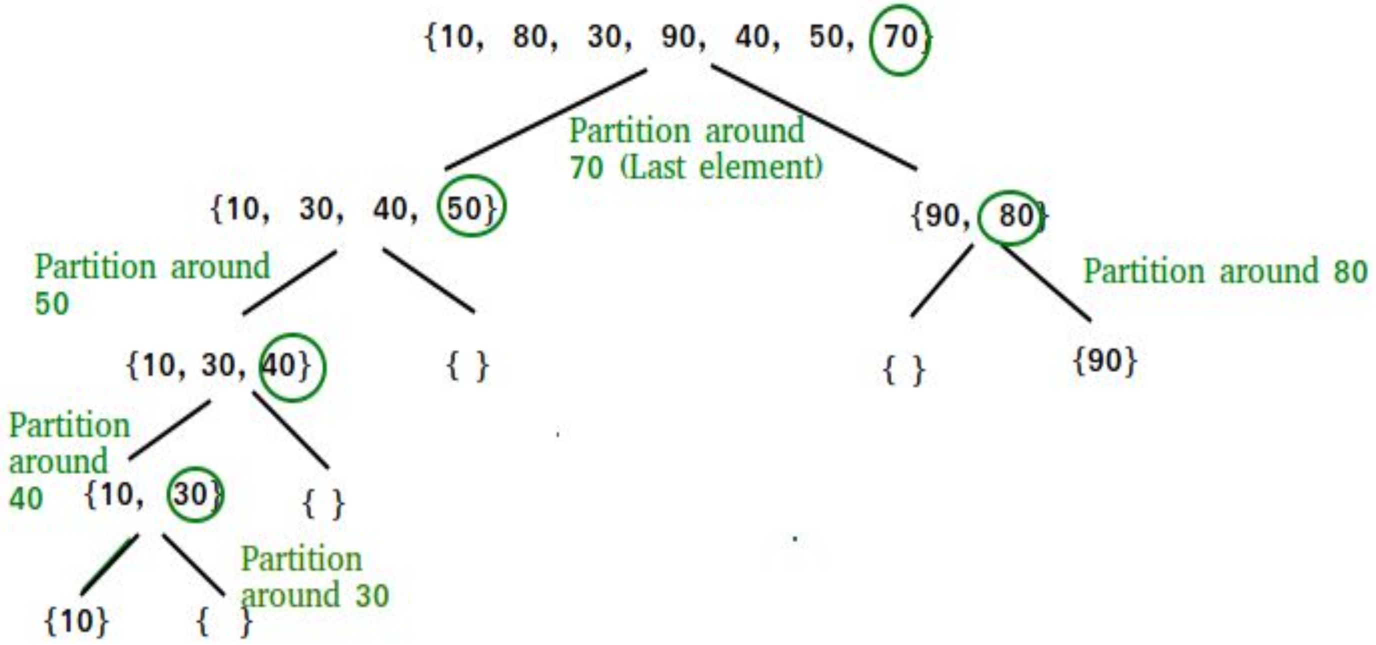
```
void merging(int low, int mid, int high) {
   int l1, l2, i;

   for(l1 = low, l2 = mid + 1, i = low; l1 <= mid && l2 <= high; i++) {
      if(a[l1] <= a[l2])
         b[i] = a[l1++];
      else
         b[i] = a[l2++];
   }

   while(l1 <= mid)
      b[i++] = a[l1++];

   while(l2 <= high)
      b[i++] = a[l2++];

   for(i = low; i <= high; i++)
      a[i] = b[i];
}
```

Quick Sort

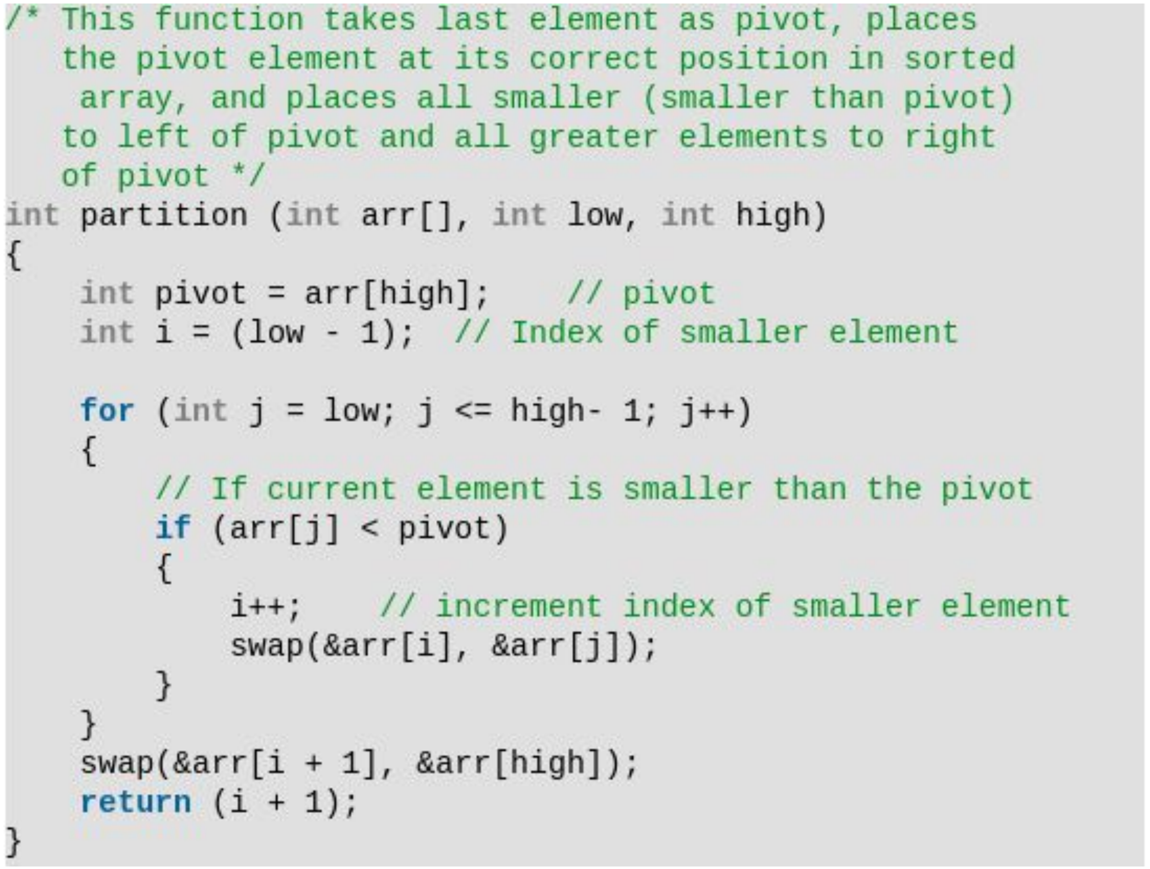
Like Merge Sort, QuickSort is a Divide and Conquer algorithm. It picks an element as pivot and partitions the given array around the picked pivot. There are many different versions of quickSort that pick pivot in different ways.

1. Always pick first element as pivot.
2. Always pick last element as pivot (implemented below)
3. Pick a random element as pivot.
4. Pick median as pivot.

The key process in quickSort is partition(). Target of partitions is, given an array and an element x of array as pivot, put x at its correct position in sorted array and put all smaller elements (smaller than x) before x, and put all greater elements (greater than x) after x. All this should be done in linear time.


{10, 80, 30, 90, 40, 50, 70}
Partition around 70 (Last element)
{10, 30, 40, 50}
{90, 80}
Partition around 50
Partition around 80
{10, 30, 40}
{ }
{ }
{90}
Partition around 40
{10, 30}
{ }
Partition around 30
{10}
{ }

```
/* The main function that implements QuickSort
 arr[] --> Array to be sorted,
  low  --> Starting index,
  high  --> Ending index */
void quickSort(int arr[], int low, int high)
{
    if (low < high)
    {
        /* pi is partitioning index, arr[p] is now
           at right place */
        int pi = partition(arr, low, high);

        // Separately sort elements before
        // partition and after partition
        quickSort(arr, low, pi - 1);
        quickSort(arr, pi + 1, high);
    }
}
```



```c
/* This function takes last element as pivot, places
   the pivot element at its correct position in sorted
    array, and places all smaller (smaller than pivot)
   to left of pivot and all greater elements to right
   of pivot */
int partition (int arr[], int low, int high)
{
    int pivot = arr[high];    // pivot
    int i = (low - 1);  // Index of smaller element

    for (int j = low; j <= high- 1; j++)
    {
        // If current element is smaller than the pivot
        if (arr[j] < pivot)
        {
            i++;    // increment index of smaller element
            swap(&arr[i], &arr[j]);
        }
    }
    swap(&arr[i + 1], &arr[high]);
    return (i + 1);
}
```

Insertion Sort

Insertion sort is a simple sorting algorithm that works similar to the way you sort playing cards in your hands. The array is virtually split into a sorted and an unsorted part. Values from the unsorted part are picked and placed at the correct position in the sorted part.

Algorithm

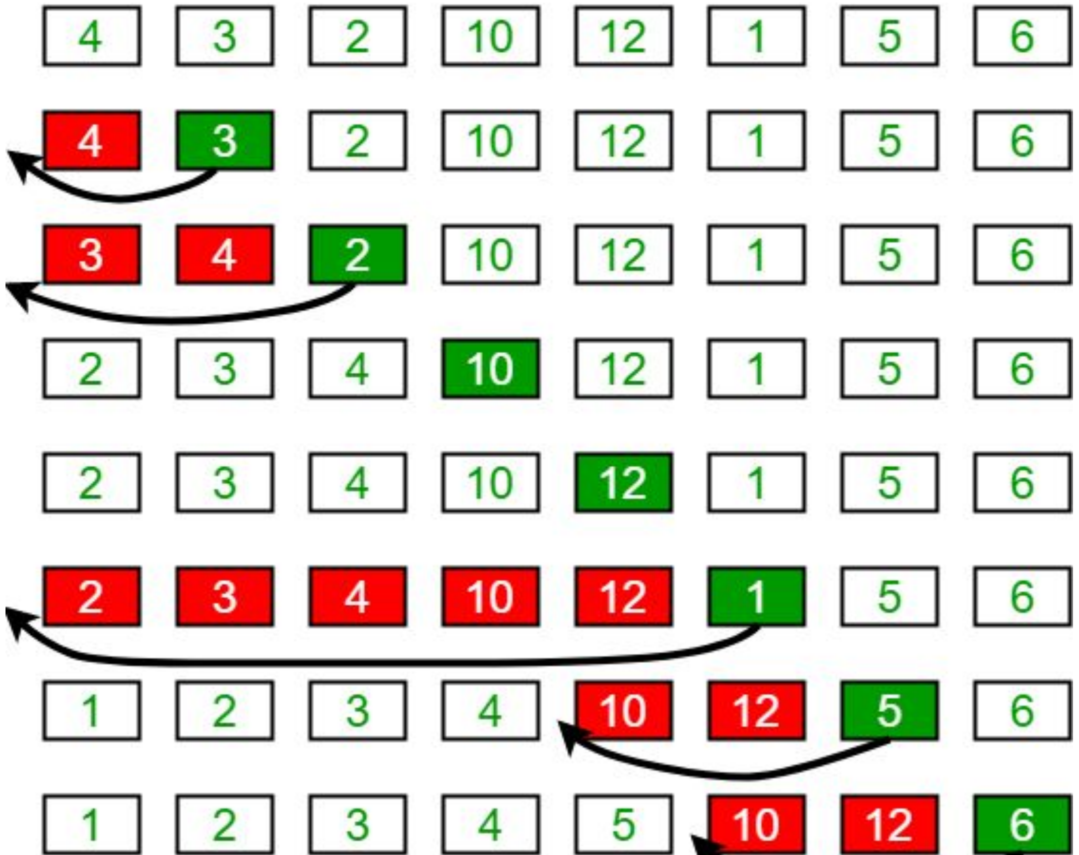
To sort an array of size n in ascending order:

1: Iterate from arr[1] to arr[n] over the array.

2: Compare the current element (key) to its predecessor.

3: If the key element is smaller than its predecessor, compare it to the elements before. Move the greater elements one position up to make space for the swapped element.

Insertion Sort Execution Example



```
/* Function to sort an array using insertion sort*/
void insertionSort(int arr[], int n)
{
    int i, key, j;
    for (i = 1; i < n; i++) {
        key = arr[i];
        j = i - 1;

        /* Move elements of arr[0..i-1], that are
          greater than key, to one position ahead
          of their current position */
        while (j >= 0 && arr[j] > key) {
            arr[j + 1] = arr[j];
            j = j - 1;
        }
        arr[j + 1] = key;
    }
}
```

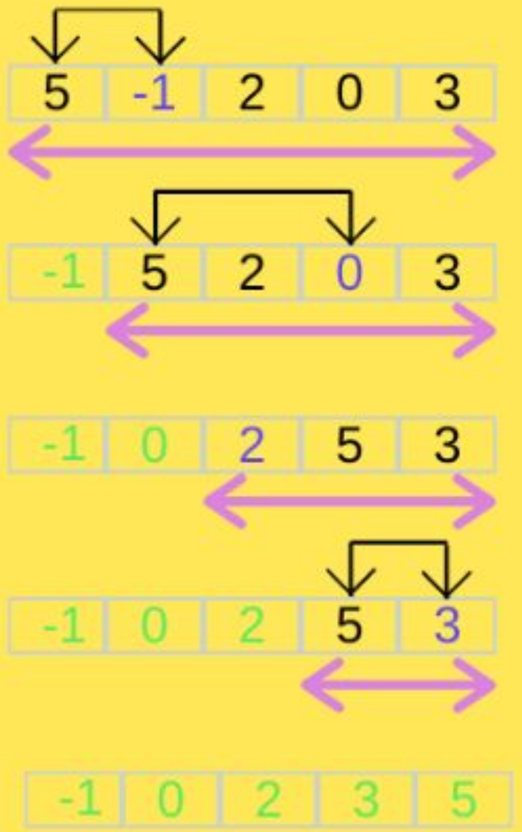
Selection Sort

The selection sort algorithm sorts an array by repeatedly finding the minimum element (considering ascending order) from unsorted part and putting it at the beginning. The algorithm maintains two subarrays in a given array.

1) The subarray which is already sorted.

2) Remaining subarray which is unsorted.

In every iteration of selection sort, the minimum element (considering ascending order) from the unsorted subarray is picked and moved to the sorted subarray.



Green = Sorted

Blue = Current minimum

Find minimum elements in unsorted array and swap if required (element not at correct location already).

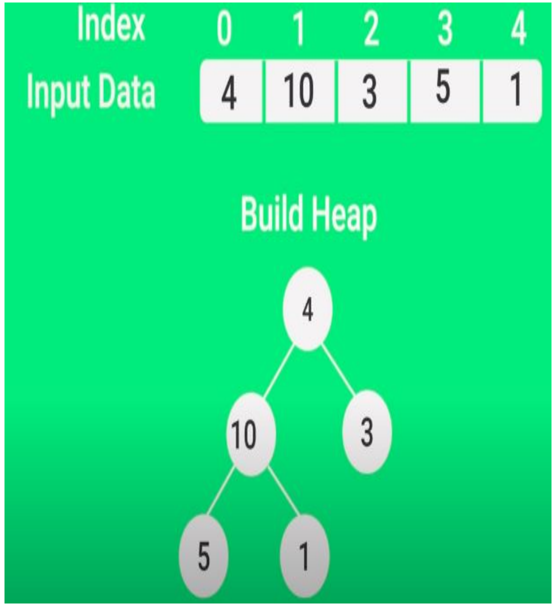
```c
void selectionSort(int arr[], int n)
{
    int i, j, min_idx;

    // One by one move boundary of unsorted subarray
    for (i = 0; i < n-1; i++)
    {
        // Find the minimum element in unsorted array
        min_idx = i;
        for (j = i+1; j < n; j++)
          if (arr[j] < arr[min_idx])
            min_idx = j;

        // Swap the found minimum element with the first element
        swap(&arr[min_idx], &arr[i]);
    }
}
```

Heap Sort

Heap sort is a comparison based sorting technique based on Binary Heap data structure. It is similar to selection sort where we first find the maximum element and place the maximum element at the end. We repeat the same process for remaining element.

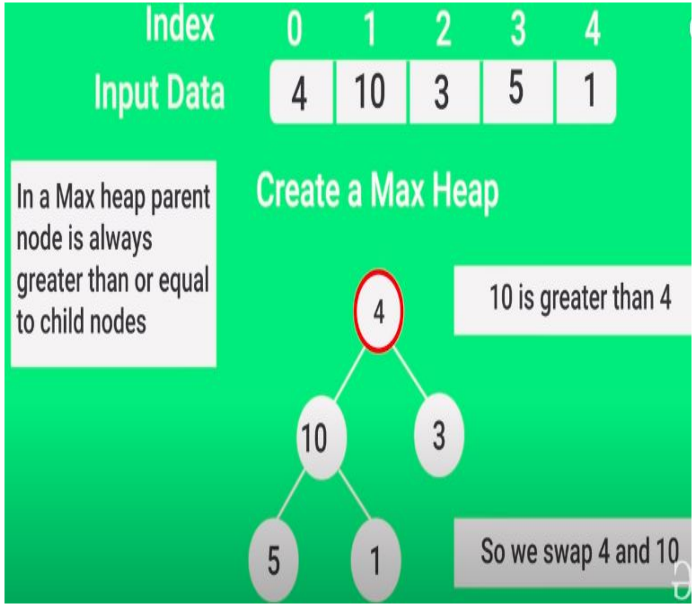
A complete binary tree is a binary tree in which every level, except possibly the last, is completely filled, and all nodes are as far left as possible (Source Wikipedia)

A Binary Heap is a Complete Binary Tree where items are stored in a special order such that value in a parent node is greater(or smaller) than the values in its two children nodes. The former is called as max heap and the latter is called min heap.



Index	0	1	2	3	4
Input Data	4	10	3	5	1

Build Heap



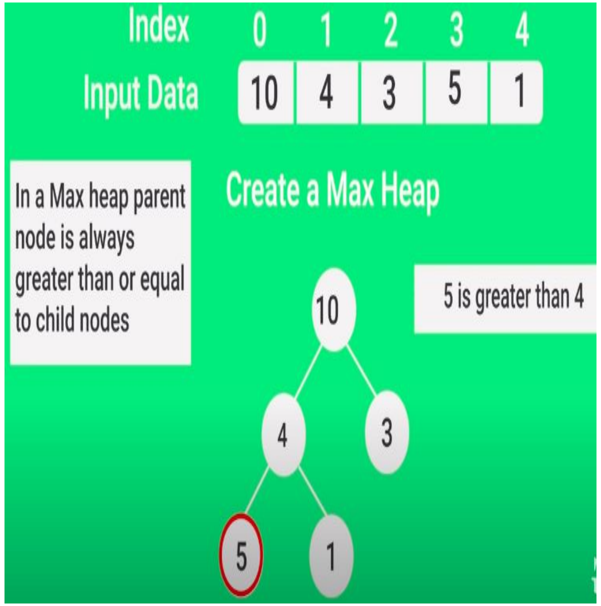
Index	0	1	2	3	4
Input Data	4	10	3	5	1

In a Max heap parent node is always greater than or equal to child nodes

Create a Max Heap

10 is greater than 4

So we swap 4 and 10

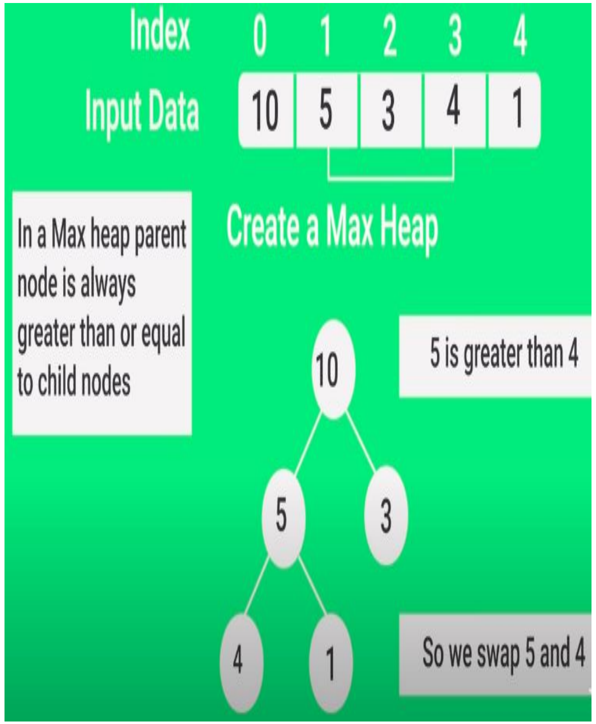


Index	0	1	2	3	4
Input Data	10	4	3	5	1

In a Max heap parent node is always greater than or equal to child nodes

Create a Max Heap

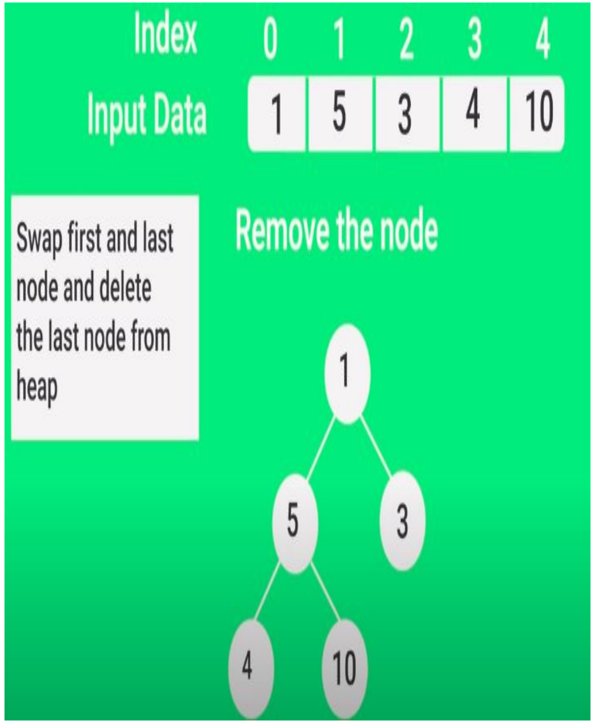
5 is greater than 4



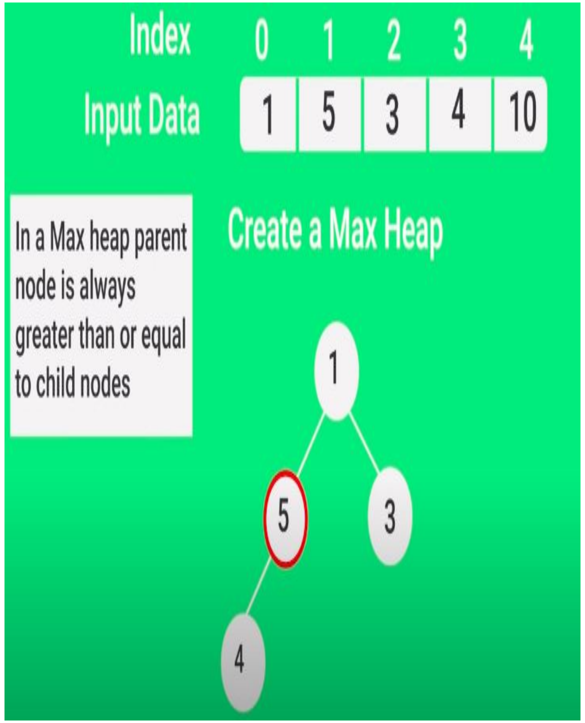
In a Max heap parent node is always greater than or equal to child nodes

5 is greater than 4

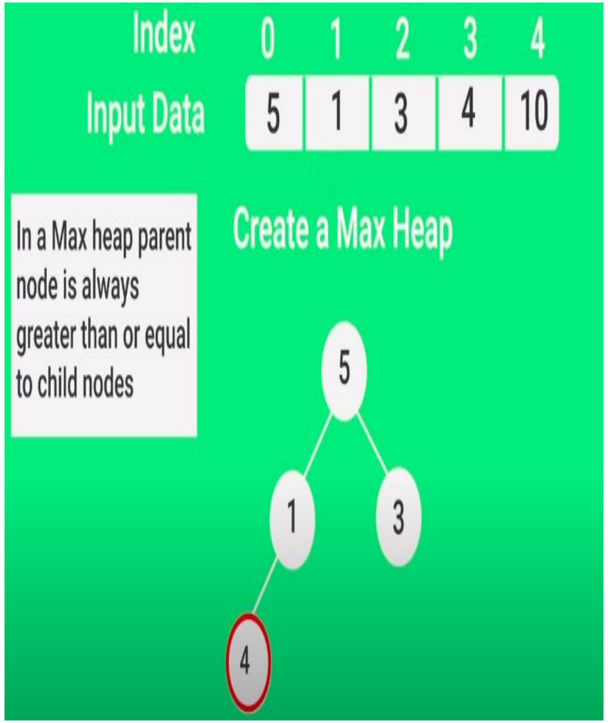
So we swap 5 and 4

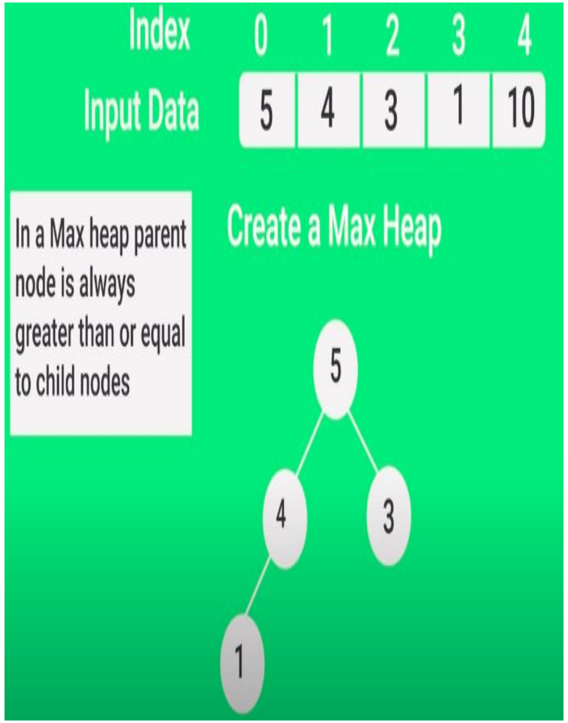


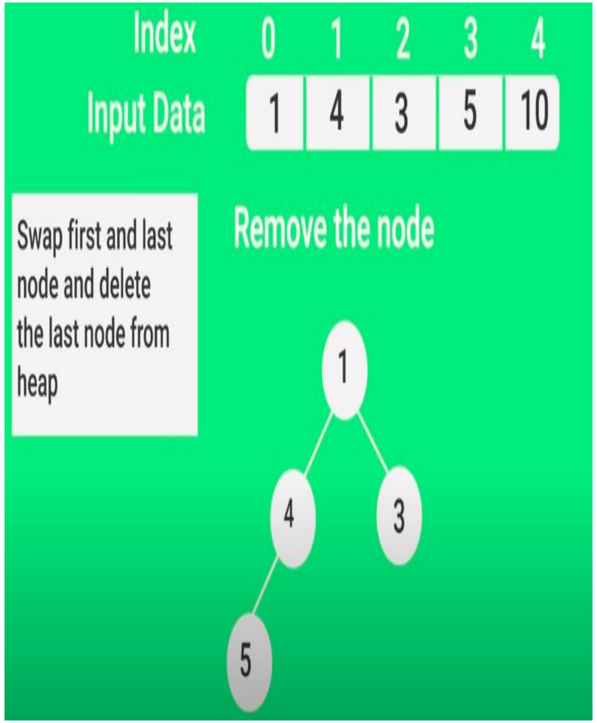
Swap first and last node and delete the last node from heap

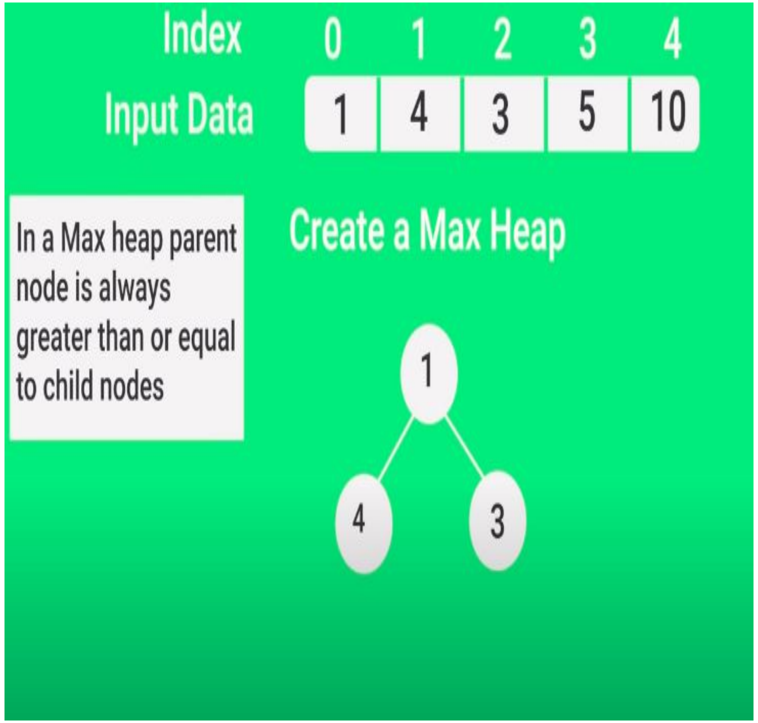


In a Max heap parent node is always greater than or equal to child nodes

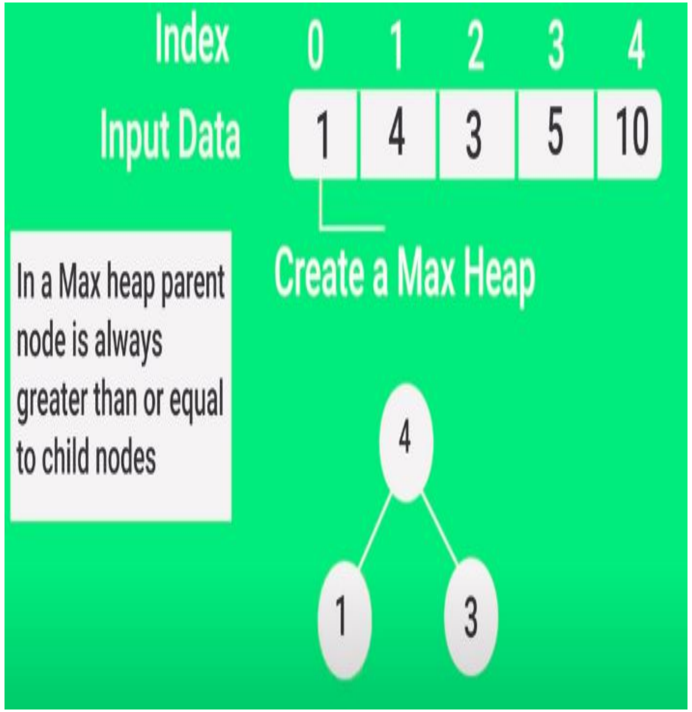

Index
0
1
2
3
4
Input Data
5
1
3
4
10
In a Max heap parent node is always greater than or equal to child nodes
Create a Max Heap
5
1
3
4


Index
0
1
2
3
4
Input Data
5
4
3
1
10
In a Max heap parent node is always greater than or equal to child nodes
Create a Max Heap
5
4
3
1

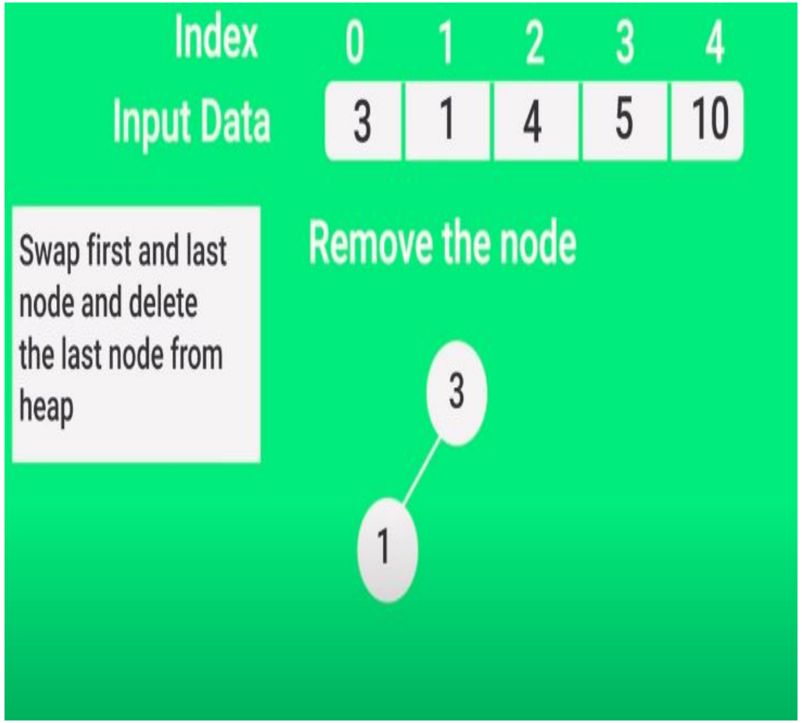

Index
0
1
2
3
4
Input Data
1
4
3
5
10
Swap first and last node and delete the last node from heap
Remove the node
1
4
3
5



In a Max heap parent node is always greater than or equal to child nodes



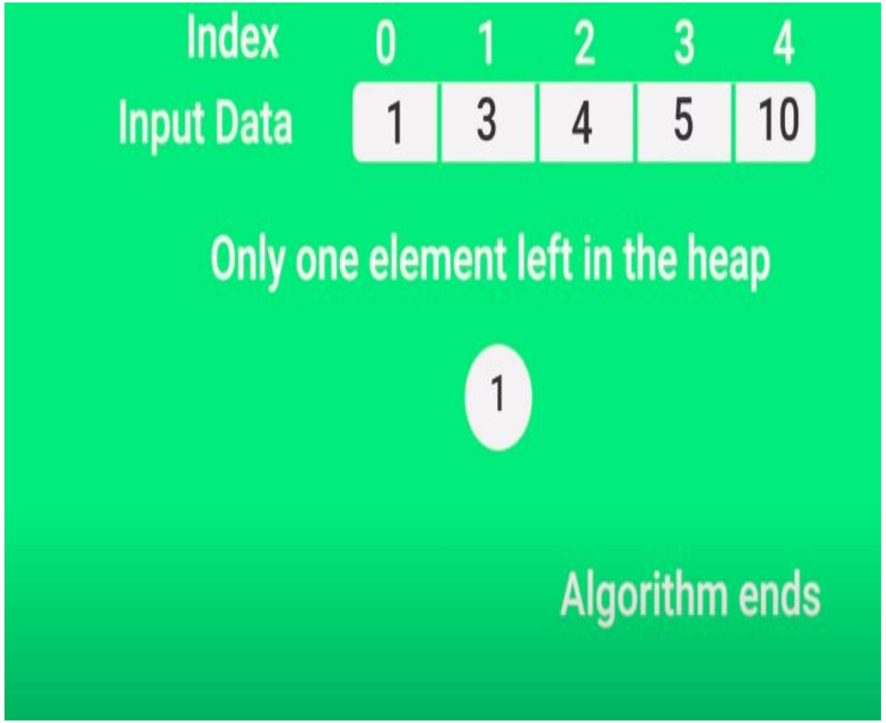
In a Max heap parent node is always greater than or equal to child nodes



Index	0	1	2	3	4
Input Data	3	1	4	5	10

Swap first and last node and delete the last node from heap

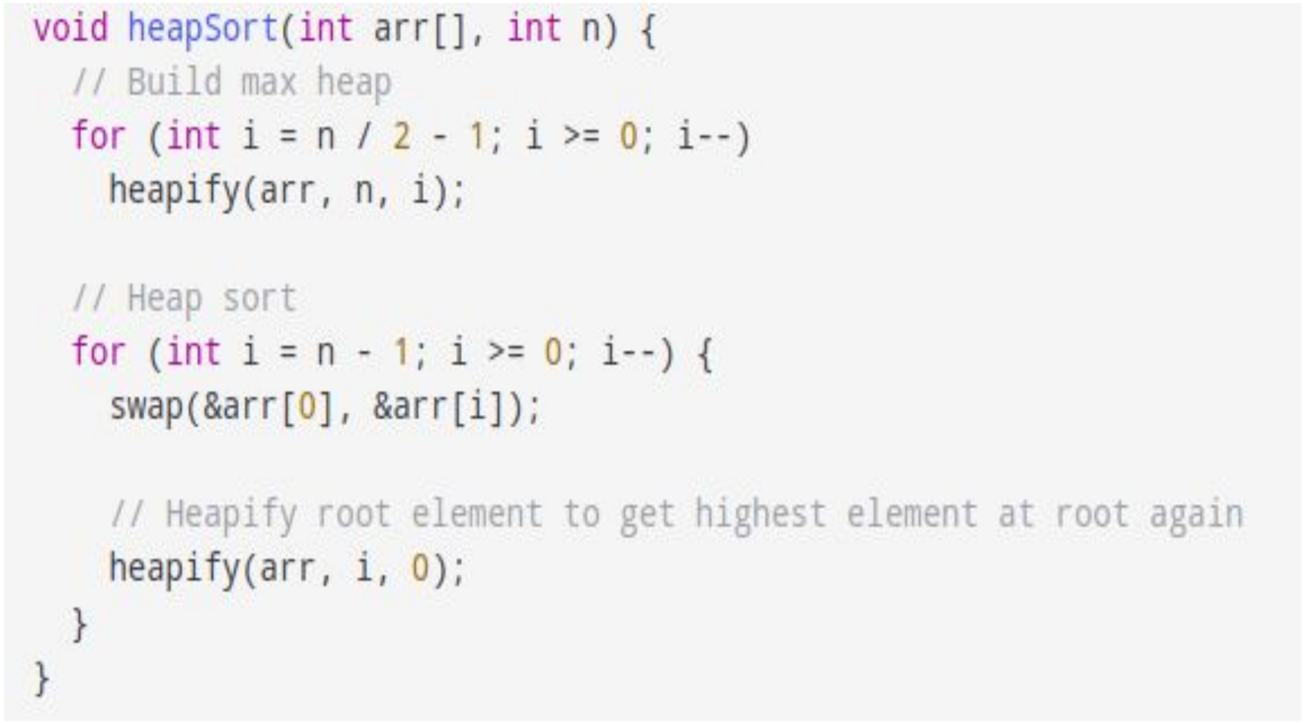
Remove the node



Index	0	1	2	3	4
Input Data	1	3	4	5	10

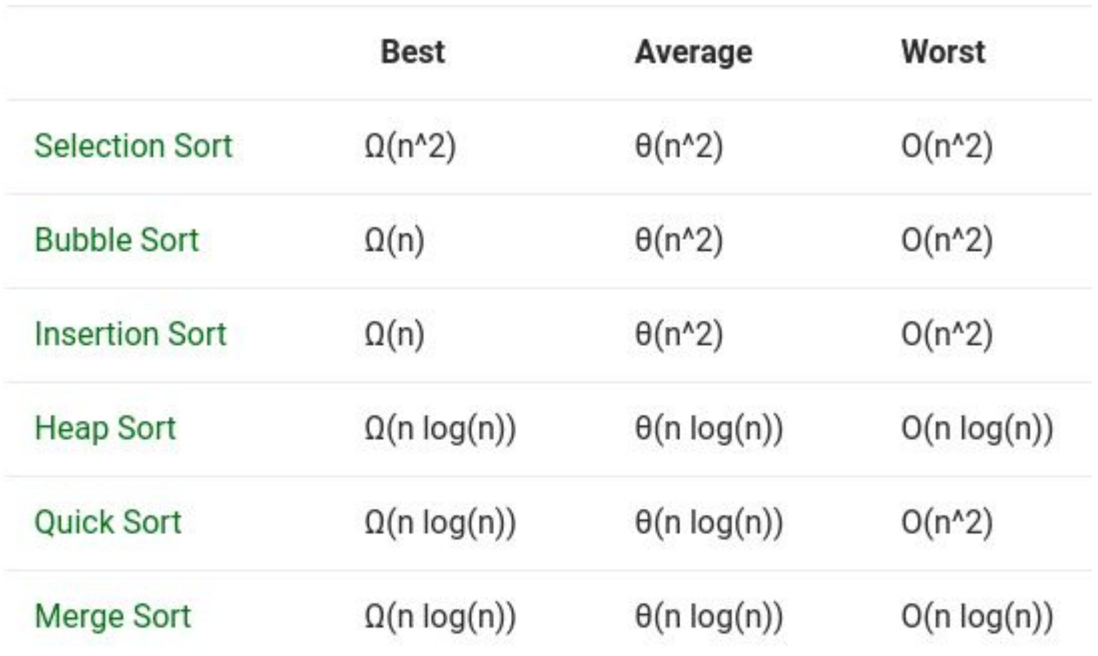
Only one element left in the heap

Algorithm ends



```c
void heapSort(int arr[], int n) {
  // Build max heap
  for (int i = n / 2 - 1; i >= 0; i--)
    heapify(arr, n, i);

  // Heap sort
  for (int i = n - 1; i >= 0; i--) {
    swap(&arr[0], &arr[i]);

    // Heapify root element to get highest element at root again
    heapify(arr, i, 0);
  }
}
```

```
void heapify(int arr[], int n, int i) {
  // Find largest among root, left child and right child
  int largest = i;
  int left = 2 * i + 1;
  int right = 2 * i + 2;

  if (left < n && arr[left] > arr[largest])
    largest = left;

  if (right < n && arr[right] > arr[largest])
    largest = right;

  // Swap and continue heapifying if root is not largest
  if (largest != i) {
    swap(&arr[i], &arr[largest]);
    heapify(arr, n, largest);
  }
}
```



	Best	Average	Worst
Selection Sort	Ω(n^2)	θ(n^2)	O(n^2)
Bubble Sort	Ω(n)	θ(n^2)	O(n^2)
Insertion Sort	Ω(n)	θ(n^2)	O(n^2)
Heap Sort	Ω(n log(n))	θ(n log(n))	O(n log(n))
Quick Sort	Ω(n log(n))	θ(n log(n))	O(n^2)
Merge Sort	Ω(n log(n))	θ(n log(n))	O(n log(n))